Rational Dynamic Loads Analysis for Air Cushion Vehicles in Random Seaway or Terrain

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A probabilistic approach is suggested as a rational method of assessing dynamic loads prevailing on an air cushion vehicle operating in a random seaway or terrain. The hull load distributions, expressed in terms of shear force, bending moment, and torsion moment, are based on a criterion of selecting an acceptable frequency of occurrence of loading determined from the mission requirements of the vehicle. The method also includes a rational approach to the problem of combining static loads with stationary random dynamic loads (seaway or terrain) and nonstationary dynamic loads (wave slamming, terrain impact).

Nomenclature

$[A]$ = inertia matrix for linear equations of motion
$[B]$ = damping matrix for linear equations of motion
$[C]$ = stiffness matrix for linear equations of motion
z/η_0 = heave transfer function
θ/η_0 = pitch transfer function
ϕ/η_0 = roll transfer function
m_B/η_0 = cushion mass transfer function
P_B/η_0 = cushion pressure transfer function
D_1 = linear wave or terrain induced sinusoidal heave force
D_2 = linear wave or terrain induced sinusoidal pitch moment
D_3 = linear wave or terrain induced sinusoidal roll moment
D_4 = linear wave or terrain induced sinusoidal cushion pumping
C_B = adiabatic stiffness $[(\bar{P}_B+P_A)\gamma/V_B]$
P_A = atmospheric pressure
ρ_A = atmospheric pressure
$\bar{P}_B$ = nominal cushion pressure
γ = ratio of specific heats for air
V_B = cushion volume
A_B = cushion area
L = cushion length
B = cushion width
m = ratio of distance from c.g. to bow to total length ℓ_b/L
ℓ_s = distance from c.g. to stern

$$I_1 = \begin{cases} (2/k_x k_y)\sin(k_y B/2)(\Psi - i\Phi), & k_x k_y \neq 0 \\ (2L/k_y)\sin(k_y B/2), & k_x = 0, k_y \neq 0 \\ BL, & k_x = k_y = 0 \\ (B/k_x)(\Psi - i\Phi), & k_x \neq 0, k_y = 0 \end{cases}$$

m_c = uniform distribution part of mass of craft per unit length
W_{sj} = discrete weights at coordinates (x_j, y_s)
$K_{\theta D}$ = distributed pitch spring rate derived from model tests
$K_{\phi D}$ = distributed roll spring rate derived from model tests
K_θ = total pitch spring rate = $K_{\theta D}\frac{1}{3}L^3(1-3m+3m^2)$
K_ϕ = total roll spring rate = $K_{\phi D}(B^3/12)$
i = $(-1)^{1/2}$, index
z_s = static heave position due to craft weight
θ_s = static pitch position due to craft weight
ϕ_s = static roll position due to craft weight
P_{BS} = static cushion pressure due to craft weight
R_1 = fan conductance $R_1 = F_c - (\bar{Q}_f/2\bar{P}_B)$
$\delta(x_j)$, $\delta(y_s)$ = unity at $x=x_j$ or $y=y_s$. Zero for all other x_j, y_s
F_c = fan characteristic slope $(\partial\bar{Q}_f/\partial\bar{P}_B)$
G = heave dependent leakage parameter $G = C_D C(2P_B/\rho_A)^{1/2}$
C = craft leakage perimeter
C_D = leakage discharge coefficient
U = craft forward speed
χ = craft heading angle
$V_s(x), V_s(y)$ = longitudinal and transverse distributed static shear
$M_{BS}(x)$, $M_{BS}(y)$ = longitudinal and transverse distributed static bending moment
$L(n)$ = load level for long-term probability calculation (in loads)
$f_L(h,n)$ = frequency of occurrence of $L(h)$
$i1, i2, i3$ = mission profile condition indices for off-cushion water, on-cushion water, and on-cushion terrain, respectively
$j1, j2, j3$ = heading angle condition indices for off-cushion water, on-cushion water, and on-cushion terrain, respectively
$k1, k2, k3$ = weight condition indices for off-cushion water, on-cushion water, and on-cushion terrain, respectively
$\sigma_{nofw}^{(i1,j1,k1)}$ = rms load for off-cushion water for condition described by (i1, j1, k1)
$\sigma_{nonw}^{(i2,j2,k2)}$ = rms load for on-cushion water for condition described by (i2, j2, k2)

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$\sigma_{nont}^{(i3,j3,k3)}$ = rms load for on-cushion terrain for condition described by (i3, j3, k3)
$f_{eofw}^{i1,j1,k1}$ = average response frequency of $\sigma_{nofw}^{(i1,j1,k1)}$
$f_{eonw}^{(i2,j2,k2)}$ = average response frequency of $\sigma_{nonw}^{(i2,j2,k2)}$
$f_{enot}^{(i3,j3,k3)}$ = average response frequency of $\sigma_{nont}^{(i3,j3,k3)}$ for on-cushion terrain

$P_{1ofw}^{(i1)}$, $P_{1onw}^{(i2)}$, $P_{1ont}^{(j1)}$ = probability of mission profile condition for off-cushion water, on-cushion water, and on-cushion terrain, respectively

$P_{2ofw}^{(j1)}$, $P_{2onw}^{(j2)}$, $P_{2ont}^{(j3)}$ = probability of craft heading angle condition for off-cushion water, on-cushion water, and on-cushion terrain, respectively.

$P_{3ofw}^{(k1)}$, $P_{3onw}^{(k2)}$, $P_{3ont}^{(k3)}$ = probability of craft weight condition for off-cushion water, on-cushion water, and on-cushion terrain, respectively

P_{off} = probability of off-cushion condition
P_{on} = probability of on-cushion condition
P_w = probability of craft operating over water
P_T = probability of craft operating over terrain
K_σ = factor times the rms level for Gaussian probability
σ_i, σ_i' = rms relative displacement and velocity, respectively, between ith bow-slamming station and wave surface

Introduction

THE rational design of high-performance air cushion vehicles requires methods whereby the vehicle structural loads may be assessed to assure structural adequacy, but not over conservatively. The random nature of the seaway or terrain environment is expressed in terms of Power Spectral Density (PSD) of wave or terrain amplitude. Thus a frequency domain representation of the craft dynamics, and the utilization of input-output techniques is the most direct method of dynamic motion and loads analysis.

In this paper, the rigid body longitudinally distributed shear force, bending moment, and torsion moment, and the transversely (laterally) distributed shear force and bending moment loads prevailing on the hull of an air cushion vehicle are determined from the linearized equations of motion in the frequency domain for the vehicle operating in random seaway or terrain. The loads transfer functions are expressed in terms of the vertical plane dynamic transfer functions of craft motion, and the root mean square dynamic loads are computed by power spectral methods using a digital computer program developed for that purpose. A rational design loads criterion is developed whereby the long term probability loadings[3] are determined from seaway or terrain roughness vs speed requirements imposed by specific missions. The loads resulting from two such mission types are compared: 1) the Amphibious Assault Landing Craft (AALC) mission and 2) the Arctic Surface Effect Vehicle (ASEV) mission. These loadings consist of both stationary (seaway or terrain) components and nonstationary (quasi-static slamming and structural dynamic slamming) components. For a preselected design acceptable frequency of occurrence, these components are combined statistically. The resulting loads are considered the design limit loads for long term operation in accordance with the mission requirements.

Not all of the design loads are included in the present paper, but only those of significance in evaluating overall hull bending and torsion capabilities. Localized or distributed pressure forces important to the design of hull panel plating are not considered, nor are loads induced by onboard components or auxilliary equipment. The basic application of the methods to be discussed is in the preliminary structural design phases of an air cushion vehicle, where the overall dimensions and section moduli are to be determined.

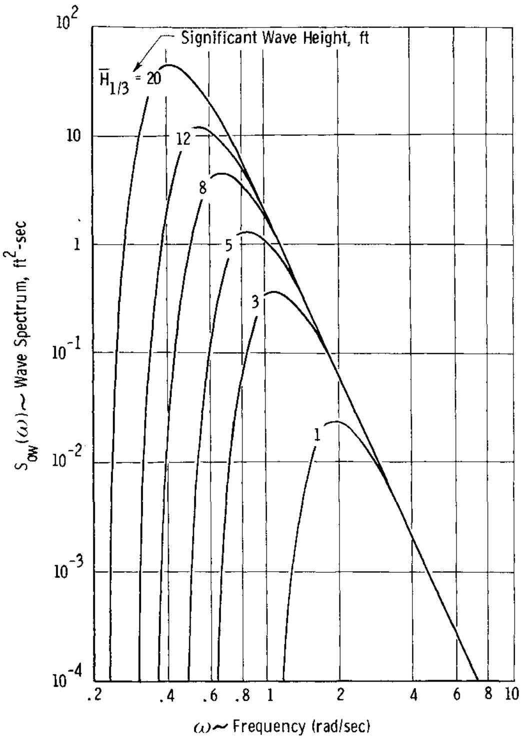

Fig. 1 Pierson-Moskowitz storm sea spectra.

II. Analytical Procedure

A. Load Categories

There are two basic load categories to consider for air cushion vehicles: 1) those that are random and 2) those that are deterministic. The random load category includes all loads induced by traversing a seaway or terrain that may be described stochastically, as well as wave slamming loads that, although nonstationary stochastically, have a random description as to frequency of occurrence. The deterministic load category includes loads that are more easily described by discrete, deterministic, situations such as landing, emergency stops, static load distributions, impact of specified obstacles or ice ridges, and the like. A rational loads analysis will attempt to assess the implications of both kinds of loads on the design, and whether or not certain classes of stationary and nonstationary loads should be combined. The approach in this paper is to present a method of rational assessment and combination of loads such that the craft will be designed with specified, quantitative, mission requirements in mind, and not arbitrarily selected "worst-case" conditions.

B. Environment Descriptions

The random description of the seaway is usually given in terms of a power spectral density of wave amplitude, or energy spectrum. Figure 1 shows one such description based on a mathematical representation of actual spectra derived from seaway measurements, the Pierson-Moskowitz spectrum. This spectrum is widely used to describe "fully arisen" storm seas in deep water. Figure 2 shows a remarkably different type of spectrum descriptive of terrain or ice ridge heights found in the Arctic. This particular distribution is derived from measurements made by the U.S. Army Cold

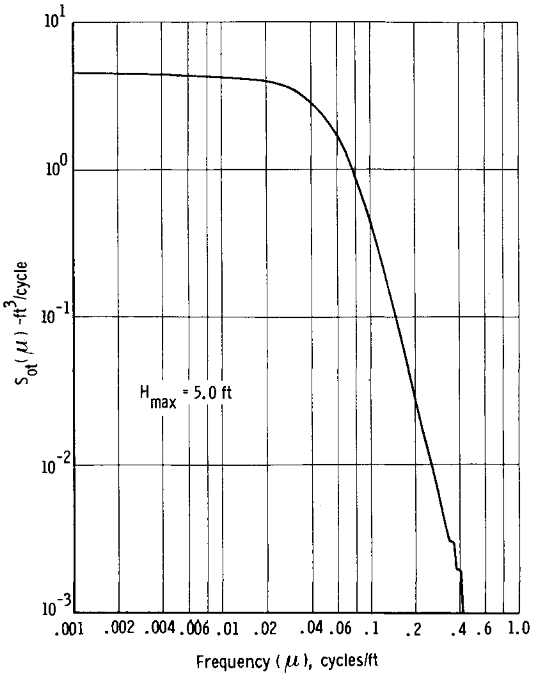

Fig. 2 Terrain spectrum.

Regions Research and Engineering Laboratory (CCREL),[1] where ridge heights were found to be distributed by a truncated Gaussian distribution, and ridge spaces were found to be distributed by an exponential distribution. The power spectral density shown in Fig. 2 combines these statistical distributions and expresses the ridge height density as a function of spatial frequency.

The complex wave form representative of either the overland surface or the sea is characterized as given in Ref. 2 by

$$\eta(x, y, t) = \eta_0 \, e^{-i(k_x x + k_y y - \omega_e t)} \tag{1}$$

The previous expression assumes a wave-craft interaction geometry as shown in Fig. 3 taken from Ref. 2. The wave surface frequency numbers k_x and k_y differ for overland surface (terrain and seaway). For seaway they are given by

$$k_x = (\omega^2/g) \cos\chi, \; k_y = (\omega^2/g) \sin\chi \tag{2}$$

with ω_e, the encounter frequency, a function of wave frequency was

$$\omega_e = \begin{cases} (1-\alpha)\,\omega, \; \alpha<1 \\ -(1-\alpha)\,\omega, \; \alpha>1 \end{cases}, \; \alpha = (U\omega/g)\cos\chi \tag{3}$$

For terrain, k_x and k_y are given by

$$k_x = 2\pi\mu \cos\chi, \; k_y = 2\pi\mu \sin\chi \tag{4}$$

with ω_e, the encounter frequency, a function of terrain spatial frequency μ as

$$\omega_e = -2\pi U\mu \cos\chi, \; 90° < \chi \leq 180° \tag{5}$$

C. Craft Loads Descriptions

The foregoing descriptions expressing the loading environment in the frequency domain strongly suggest using a

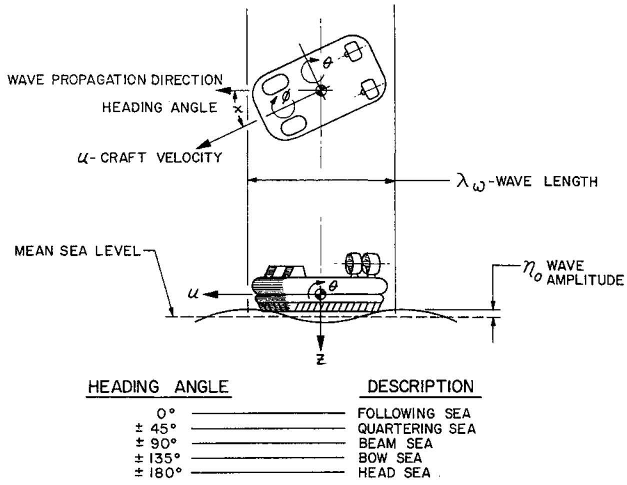

Fig. 3 Description of wave and craft geometry.

corresponding description of the craft dynamic load parameters as transfer functions or response amplitude operators (square of the amplitude of the transfer functions) between the load parameter of interest and the wave or terrain amplitude. In Ref. 2 the frequency domain mathematical model of an ACV in the heave, pitch, roll, and cushion flow degrees of freedom was derived and expressed as follows

$$\left[-\omega_e^2 [A] + i\omega_e [B] + [C] \right] \begin{Bmatrix} z/\eta_0 \\ \theta/\eta_0 \\ \phi/\eta_0 \\ m_B/\eta_0 \end{Bmatrix} = \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix} \tag{6}$$

with the auxiliary cushion pressure equation given by

$$\frac{P_B}{\eta_0} = \frac{C_B}{\rho_A}\frac{m_B}{\eta_0} + C_B A_B \frac{z}{\eta_0} - C_B A_B (m-0.5)$$

$$L\theta/\eta_0 + C_B I_1 \tag{7}$$

1) *Dynamic Loads Transfer Functions*

The craft loads model is developed utilizing the solutions of Eq. (6) in the frequency domain, and basic beam theory, to derive transfer functions of longitudinally distributed shear force, bending moment, and torsion moment. These loads transfer functions were derived from Eqs. (18-33) in Ref. 2 utilizing the substitution of $(i\omega_e)^\eta$ for the ηth derivative with respect to time. The details of the resulting expressions are not included here for brevity. The craft coordinate system is right hand Cartesian, with origin at the craft c.g. and x positive forward, y positive starboard, and z positive down. Rotations are positive in the sense of a right hand screw progressing in the positive coordinate directions. (See Fig. 3.)

2) *Static Loads Equations*

Consistent with the dynamic loads transfer functions craft model are corresponding loads equations describing the static equilibrium of the craft under its own weight

a) *Longitudinally distributed static loads*

Shear force

$$V_s(x) = m_c g(x+\ell_s) + \sum_{j=1}\sum_{s=1} W_{sj}\delta(x_j)$$

$$-K_{\theta D}(x+\ell_s)z_s + K_{\theta D}\frac{(x^2-\ell_s^2)}{2}\theta_s - \frac{A_B}{L}(x+\ell_s)P_{BS} \tag{8}$$

Bending moment

$$M_{BS}(x)=m_c g\,\frac{(x+\ell_s)^2}{2}+\sum_{j=1}^{i}\sum_{s=1}^{m}W_{sj}(x-x_j)\delta(x_j)$$
$$-K_{\theta D}\,\frac{(x+\ell_s)^2}{2}\,z_s+K_{\theta D}\,\frac{(x+\ell_s)^2(x-2\ell_s)}{6}\,\theta_s$$
$$-\frac{A_B}{L}\,\frac{(x+\ell_s)^2}{2}\,P_{BS} \tag{9}$$

b) Transversely distributed static loads

Shear force

$$V_s(y)=\frac{m_c gL}{B}\left(y+\frac{B}{2}\right)+\sum_{s=1}^{r}\sum_{j=1}^{n}W_{sj}\delta(y_s)$$
$$-k_{\phi D}(y+B/2)z_s+\frac{K_{\theta D}}{B}\,\frac{(\ell_b^2-\ell_s^2)}{2}\,(y+B/2)\theta_s$$
$$-K_{\phi D}\,\frac{(y^2-B^2/4)}{2}\,\phi_s-\frac{A_B}{B}\,(y+B/2)P_{BS} \tag{10}$$

Bending moment

$$M_{BS}(y)=\frac{m_c gL}{B}\,\frac{(y+B/2)^2}{2}$$
$$+\sum_{s=1}^{r}\sum_{j=1}^{n}W_{sj}(y-y_s)\delta(y_s)-K_{\phi D}\,\frac{(y+B/2)^2}{2}\,z_s$$
$$+\frac{K_{\theta D}}{B}\,\frac{(\ell_b^2-\ell_s^2)}{2}\,\frac{(y+B/2)^2}{2}\,\theta_s$$
$$-K_{\phi D}\,\frac{(y+B/2)^2(y-B)}{6}\,\phi_s$$
$$-\frac{A_B}{B}\,\frac{(y+B/2)^2}{2}\,P_{BS} \tag{11}$$

D. Long-Term Seakeeping Loads

The method which utilizes probability theory to predict general or long-term trends in structural loading is outlined in Ref. 3. Basically, the method accounts for the expected operational environment of the craft and its weight configuration. Probabilities are assigned to various conditions of sea state, or terrain state and craft speed together with heading angle and weight condition.

The basic assumption used to establish long term structural loading trends in the analysis here is that the seaway or terrain induced structural loads (shear, bending moment, torsion moment) are described essentially by narrow band random spectra, which implies normal (Gaussian) probability distribution of the load maxima. Three basic statistical quantities are utilized in the application of the method: 1) root mean square (standard deviation of the structural load $\sigma_n(i, j, k)$; 2) average frequency of response of structural load $f_{en}(i, j, k)$, 3) probabilities associated with craft configuration and/or enviromental condition $P_m(i)$, $P_m(j)$, $P_m(k)$, where the subscript n refers to the kind of load (shear, bending moment, etc.) the subscript m refers to the kind of probability (off-cushion, on-cushion, water, or terrain), and the indices (i, j, k) refer to the craft mission envelope speed wave height point, heading angle, and craft weight condition, respectively.

$$I_{B/2}g=m_c gL\,\frac{B}{2}+\sum_{j=1}^{n}\sum_{s=1}^{m}W_{sj}\left(\frac{B}{2}-y_s\right) \tag{13}$$

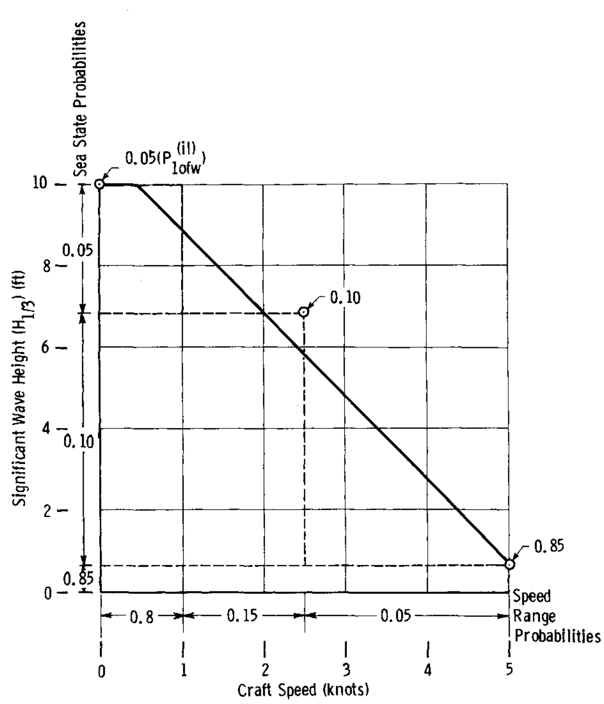

Fig. 4 AALC mission probabilities envelope off-cushion over water.

The rms load $\sigma_n(i, j, k)$ is generated from the loads transfer function and the input wave or terrain spectra by the following general relationship

$$\sigma_n(i,j,k)=\left[\int_0^{\omega_c}|H_n(i,j,k,\omega_e)|^2 S_0(\omega_e)\,d\omega_e\right]^{1/2} \tag{14}$$

where ω_c = upper cut-off frequency above which the integrand becomes negligibly small (rad/sec); $H_n(i, j, k, \omega_e)$ = load transfer function (shear, bending moment, etc.) at any point on craft ratioed to wave or terrain amplitude η_0, for a given speed-wave amplitude i, heading angle j, and weight condition k. $So(\omega_e)$ = wave or terrain amplitude spectral density ft^2/rad/sec transformed by Jacobian $(\partial\omega/\partial\omega_e)$ for waves and $(\partial\mu/\partial\omega_e)$ for terrain.

The average frequency of response of the structural load $f_{en}(i, j, k)$ is given by

$$f_{en}(i,j,k)=\frac{\left[\int_0^{\omega_c}\omega_e^2|H_n(i,j,k,\omega_e)|^2 S_0(\omega_e)\,d\omega_e\right]^{1/2}}{2\pi\sigma_n(i,j,k)} \tag{15}$$

$$\begin{bmatrix} (K_{\theta D}L+C_BA_B^2) & -(K_{\theta D}L+C_BA_B^2)(M-0.5)L & 0 & C_BA_B/\rho_A \\ (K_{\theta D}L+C_BA_B^2)\tfrac{1}{2}L & -[\tfrac{1}{3}K_{\theta D}L^3(3/2(m-1))+C_BA_B^2\tfrac{1}{2}L^2(m-0.5)] & 0 & C_BA_BL/2\rho_A \\ (K_{\theta D}L+C_BA_B^2)\tfrac{1}{2}B & -(K_{\theta D}L+C_BA_B^2)(m-0.5)\tfrac{1}{2}BL & -K_{\phi D}B^3/12 & C_BA_BB/2\rho_A \\ -\rho_A(C_BA_BR_I+G) & \rho_A(m-0.5)L(C_BA_BR_I+G) & 0 & -C_BR_I \end{bmatrix}\begin{Bmatrix} z_S \\ \theta_S \\ \phi_S \\ m_{BS}\end{Bmatrix}=\begin{Bmatrix} M_cg \\ I_{\theta B}g \\ \tfrac{1}{2}I_Bg \\ 0\end{Bmatrix} \tag{12}$$

The probabilities associated with craft configuration and/or environmental condition $P_m(i)$, $P_m(j)$, $P_m(k)$ must be derived from a rational assessment of the expected configuration and operational (mission) requirements of the craft. In terms of these quantities, the following equation expresses the expected frequency of occurrence (in occurrences per sec) $f_L(h,n)$ of loads at or above a given load level, $L(h)$

$$f_L(h,n) = P_w \Big[P_{\text{off}} \sum_{k1} P_{3ofw}^{(k1)} \sum_{j1} P_{2ofw}^{(j1)} \times \sum_{i1} P_{1ofw}^{(i1)} f_{eofw}(i1,j1,k1) \exp\Big\{ -\tfrac{1}{2}\Big[\frac{L(h)}{\sigma_{nofw}(i1,j1,k1)} \Big]^2 \Big\} + P_{\text{on}} \sum_{k2} P_{3onw}^{(k2)} \sum_{j2} P_{2onw}^{(j2)} \sum_{i2} P_{1onw}^{(i2)} f_{eonw}(i2,j2,k2) \times \exp\Big\{ -\tfrac{1}{2}\Big[\frac{L(h)}{\sigma_{nonw}(i2,j2,k2)} \Big]^2 \Big\} + P_t \sum_{k3} P_{3ont}^{(k3)} \sum_{j3} P_{2ont}^{(j3)} f_{eont}(i3,j3,k3) \exp\Big\{ -\tfrac{1}{2}\Big[\frac{L(h)}{\sigma_{nont}(i3,j3,k3)} \Big]^2 \Big\} \quad (16)$$

1) *Mission Envelope Probabilities*

The mission significant wave height vs craft speed probabilities for off-cushion $P_{1ofw}^{(i1)}$ and on-cushion $P_{1onw}^{(i2)}$ operation of the craft are shown in Figs. 4 and 5, respectively, for the AALC mission, and on-cushion $P_{1onw}^{(i2)}$ probabilities in Fig. 6 for the ASEV mission. (The loads equations for off-cushion conditions were not presented, but they were based on Lewis' transfer functions given in Ref. 4). For the ASEV mission the on-cushion terrain probabilities $P_{1ont}^{(i3)}$ were based on travel at a single craft speed over a given single terrain spectrum, so that there was no envelope for the terrain operation. This speed-terrain height condition was selected based on a mission requirement established to maximize range and minimize adverse habitability by avoiding obstacles greater than 5.0 ft.

The actual division of the mission envelopes into regions for which joint probabilities of the occurrence of a specific range of craft speed with a corresponding range of significant wave height has been accomplished by utilizing statistical oceanographic data with respect to wave height and statistical estimates of craft speed dependent on the mission craft expected duty cycle. The fineness of division of each region was somewhat arbitrarily chosen, and the joint probability for each region was assigned to the upper right hand corner of that region to insure conservatism in the determination of loads. (A point can be made that the resulting seakeeping loads are determined by the assumptions made here, and careful consideration must be given to the mission envelope required).

2) *Heading Angle Probabilities*

The craft heading angle probabilities for both off-cushion water $P_{2ofw}^{(j1)}$ and on-cushion $P_{2onw}^{(j2)}$ for both the AALC mission and the ASEV mission, and $P_{2ont}^{(j3)}$ for the ASEV mission were estimated in accordance with Table 1.

3) *Craft Weight Condition Probabilities*

For the AALC mission, only two craft weight conditions were assumed for both off-cushion and on-cushion, namely, the fully laden weight and the minimum operating weight (10% fuel remaining). For the ASEV mission only the fully laden on-cushion weight was considered for travel over water, and 2 weight conditions (75% fuel loading and 25% fuel loading) were considered to sufficiently approximate craft configuration while traversing the Arctic ice ridges. The

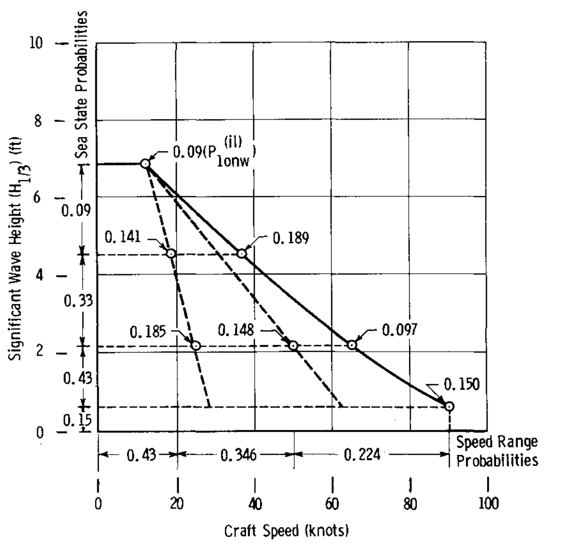

Fig. 5 AALC mission probabilities envelope on-cushion over water.

Table 1 Craft heading angle probabilities

Heading angle X (deg)	AALC Mission Off-cushion water $P(j1)$ $2ofw$	AALC Mission On-cushion water $P(j2)$ $2onw$	ASEV Mission On-cushion water $P(j2)$ $2onw$	ASEV Mission On-cushion terrain $P(j3)$ $2ont$
0 (following)-	0.35	0.35	0.35	0
90 (beam)	0.15	0.15	0.15	0
135 (bow)	0.15	0.15	0.15	0.5
180 (head)	0.35	0.35	0.35	0.5
Total	1.00	1.00	1.00	1.00

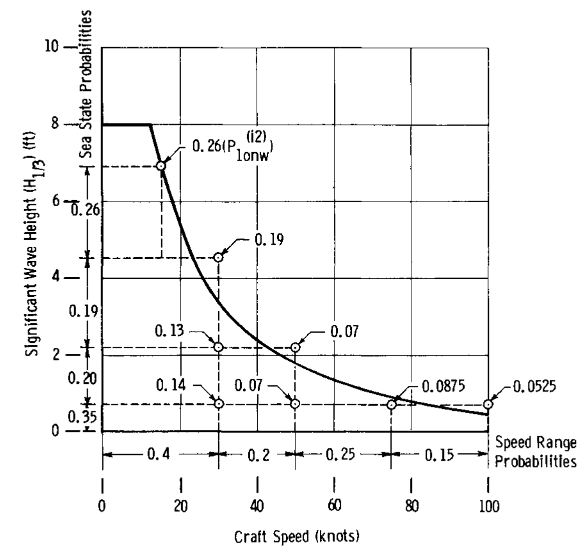

Fig. 6 ASEV mission probabilities envelope on-cushion over water.

probabilities at each of these conditions $P_{3ofw}^{(k1)}$, $P_{3onw}^{(k2)}$, and $P_{3ont}^{(k3)}$ for both the AALC and the ASEV missions are shown in Table 2.

4) *On-Cushion-Off Cushion Probabilities*

The percentages of time (probabilities) associated with on-cushion—off-cushion conditions were estimated to be $P_{\text{off}} = 0.2$, and $P_{\text{on}} = 0.8$ for the AALC mission and $P_{\text{off}} = 0.0$, and $P_{\text{on}} = 1.0$ for the ASEV mission.

Table 2 Craft weight condition probabilities

	AALC Mission		ASEV Mission	
Operating weight condition	Off-cushion water $P(k1)$ $3ofw$	On-cushion water $P(k2)$ $3onw$	On-cushion water $P(k2)$ $3onw$	On-cushion terrain $P(k3)$ $3ont$
Heavy weight	0.60	0.60	1.0	0.5
Light weight	0.40	0.40	0	0.5
Total	1.00	1.00	1.00	1.00

5) Water vs Terrain Probabilities

Since no random terrain conditions were envisioned for the AALC mission, the probability of operation over water P_w was taken to be 1.0 and the probability of operation over random terrain P_t was taken to be 0.0. For the ASEV mission, however, the craft was to be designed for most of its operating life over arctic random terrain, and thus P_w was taken to be 0.2, and P_t was taken to be 0.8.

6) Method of Applying Long Term Probability Equation

For the purpose of brevity, only the longitudinal bending moment and torsion moment, together with the transverse (lateral) bending moment, all at the craft mid-section, were utilized to establish long term trends for seaway or terrain induced loads. Figure 7 shows the long term trends of these loads for the AALC mission, and for the ASEV mission. These long term loads result from an application of Eq. (16) to the rms loads found at the mid-section as determined by the loads transfer functions utilized in a digital computer program employing Eqs. (14) and (15).

An acceptable frequency of occurrence of a given load level must be selected from the long term load trends curves. Criteria for this selection are notably lacking in current ship design practice because probabilistic load determination for design purposes is relatively new to the industry. Mansour[5] presents a very comprehensive and rational probabilistic procedure for ship design and Ochi[6] has presented an interesting paper on prediction of extreme values which are needed for rational design. However these procedures need to be incorporated to some degree into military ship design specifications for use by the industry. Military aircraft specifications, by contrast, have been quite definitive as to the acceptable frequency of occurrence of loads at or above a given load level. In particular, MIL-A-8866 (ASG), as applied to utility aircraft require that 100% limit loads not be exceeded oftener than 20 times per thousand hours of operation. This gives an acceptable frequency of occurrence of a given load once in every 50 hrs of operation. This criteria has been used in the analyses of the present paper. The actual loads distributions for the 1/50 hr frequency of occurrence were then obtained by applying appropriate K_σ factors to the rms dynamic load distributions for the most frequent (or, alternatively most severe) environmental condition specified in the mission profiles of Figs. 5 and 6.

E. Slamming

1) Slamming Frequency of Occurrence and Force

The frequency of occurrence of slams on a given bow location h_{ci} at or above a given threshold relative slamming velocity v_{si} was given in Ref. 2 as

$$FSB_i = \frac{\omega_i}{2\pi} \exp\left\{ -\frac{1}{2} \frac{1}{\sigma_i^2} \left[h_{ci}^2 + \left[\frac{v_{si}}{\omega_i} \right]^2 \right] \right\} \quad (17)$$

The actual slamming force generated at a point h_{ci} on the bow due to a given v_{si} was determined by a very detailed force

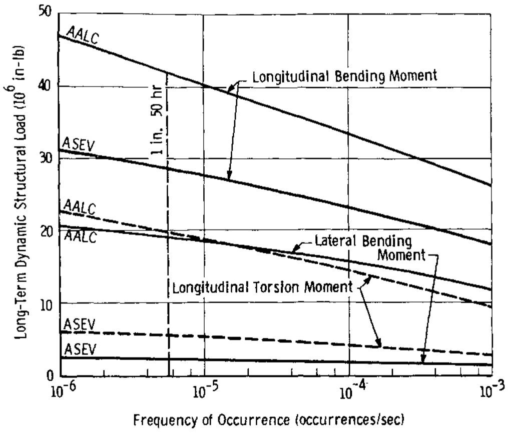

Fig. 7 AALC & ASEV mission long-term craft dynamic structural load at midsection.

and motion analysis of the craft trajectory as it comes into contact with a (assumed sinusoidal) wave of appropriate height and length having the same frequency of occurrence in a Pierson-Moskowitz sea as for the slam. Given the initial downward relative velocity and bow position at the specified craft speed, slam forces on the rigid body craft bow were determined by a time domain solution of a six-degree-of-freedom mathematical model. At each step in time, the position of the ship relative to the local water surface was determined. When contact or immersion is detected, water splash-up on the bow is computed as a function of position on the ship. Thus, wetted areas are determined for each finite slice or segment of the bow.

Peak pressures were calculated for each finite bow element slice as a function of the velocity of the stagnation line moving over the bow. The velocity of the stagnation line is a complex function of bow geometry, local water surface geometry, ship trajectory, and associated rates. Pressure distribution over the wetted finite slice was accomplished by accounting for local dead-rise angle of the slice and peak pressure. Pressure distribution and wetted area were integrated over the slice to develop both force and centroidal location. These slice forces were then summed and employed in the six second-order equations of motion of the craft. For a detailed discussion of the computational procedure and theory, see Refs. 7-9.

2) Long Term Trends

The peak vertical slamming forces on the craft bow, computed as previously described at each craft mission profile condition, heading angle, and craft weight corresponding to given frequencies of occurrence from Eq. (17) were computed and weighted in accordance with Eq. (16). The resulting curve of slamming force vs frequency of occurrence represents the long term slamming force distribution and was derived on the same basis as for the randon seaway or terrain loads. For the AALC mission this curve is shown in Fig. 8.

For asymmetric slamming the long term induced roll acceleration was also computed from the six DOF model and weighted for mission profile, heading angle, and craft weight in the same way. For the AALC mission this curve is shown in Fig. 9. This curve was used to determine the torsion loads induced on the craft due to asymmetric slamming.

Water slamming loads for the ASEV mission were found to occur so rarely that they become insignificant on a long term probability basis. The same was true for the random terrain slamming. This is a result of using a much higher skirt for the ASEV vehicle (nearly twice that for the AALC mission). Thus, the only slamming-type loads considered for the ASEV craft were from assumed ice impact occurring, due to possible failure of an on-board obstacle detection system. For the present paper these loads are not discussed, as they are con-

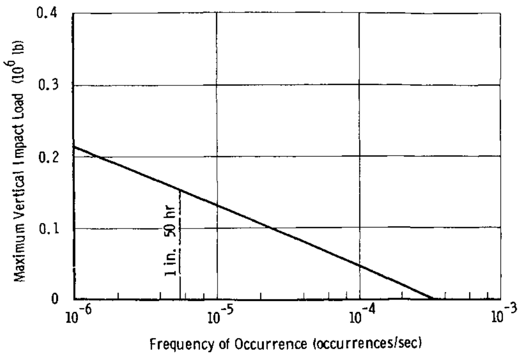

Fig. 8 Long-term vertical slamming load AALC mission.

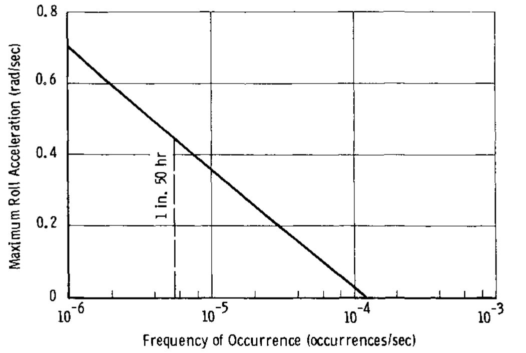

Fig. 9 Long-term roll acceleration due to asymmetric slamming.

ditions that are assumed to occur separately to the random seaway or terrain induced loads. The ice impact loads were utilized in the design of crushable structure used as an impact limiter for the craft only for the ASEV mission. Craft basic hull structure was not designed by ice impact conditions.

The longitudinal distributions of maxima of shear force, bending moment, and torsion moment due to the 1/50 hr slam load were determined by integration of the slam force or moment balanced inertially by rigid body heave, pitch, and roll accelerations. These load distributions constitute what is known as the "quasi-static" component of slam loading.

3) *Structural Dynamic Slamming Loading*

The structural dynamic longitudinal distributions of shear force, bending moment, and torsion moment were calculated, assuming that the craft may be approximately represented as a uniform free-free beam. The loads were computed in each of the first 4 bending modes, and each of the first 5 torsion modes by the shock spectrum technique. References 10 and 11 discuss this technique in detail. The shock loading pulse was obtained as the 1/50 hr "design" load for symmetric and unsymmetric loading previously discussed. The shape of the shock impulse was determined to approximate a half-sine form of duration $\tau = 0.09$ sec and having a maximum amplitude of 156,000 lb of force in bending and 5,400,000 in.-lb of torsion moment for the AALC mission (Fig. 10).

Recently, evidence has indicated that the actual shape of bow slamming pulses is probably more closely represented by a pulse that has a very short rise time to peak and a slower decay time to zero. However, according to Ref. 10 if the shock amplification spectrum is used to calculate response, the actual shock spectrum is relatively insensitive to the shape of the shock pulse. This is, of course, true only if the area under the shock pulse is the same for each differently shaped pulse. Because of this insensitivity, and because the half-sine spectrum was readily available,[11] the shock response structural dynamic loads were computed based on the aforementioned half-sine pulse, as shown in Fig. 10. Maximum envelope (maxima) modal loads result from using this well-known shock spectrum.

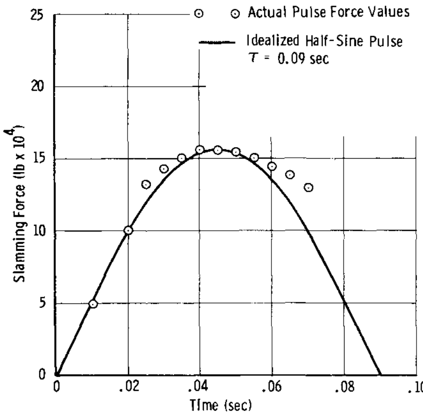

Fig. 10 Design vertical slamming force pulse applied at bow.

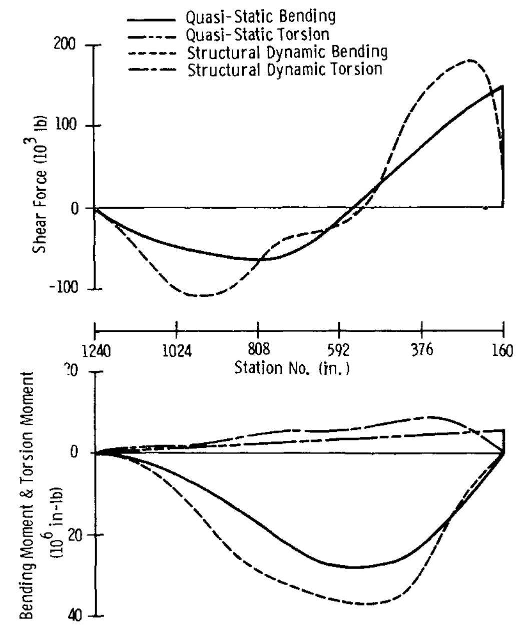

Fig. 11 Longitudinal distributed loads due to 1/50 hr slamming half-sine slamming pulse AALC mission.

The details of the method for calculating longitudinal shear force, bending moment, and torsion moment in each mode are not presented here, as this is standard for uniform beams.[11] The modes were superposed in a realistic but conservative manner. The total maximax shear load, bending moment, and torsion moment were computed by directly adding to maximax modal components accounting for the algebraic sign due to distribution in each mode. This procedure assumes that all modal peaks are in phase as they occur. Only the more severe fully laden craft configuration was used. The resulting total structural dynamic longitudinal shear force, bending moment and torsion moment for the AALC mission are shown in Fig. 11.

F. Rationale For Combining Loads

The several components of shear force, bending moment, and torsion moment, consisting of static, quasi-static slamming, structural dynamic slamming, and seakeeping sources, must be combined in a reasonable but conservative manner to establish total combined limit load conditions. The static component is deterministic and is present at all times. All of the other load components are statistical in nature.

The quasi-static component of slamming force magnitude has a probability density function that consists of a Dirac function at zero loading of value 0.75 which represents the percentage of time during which no slamming occurs, and the slam velocity is below the "threshold" slam velocity. The

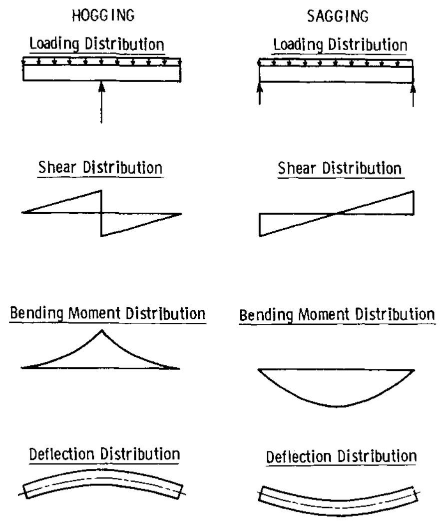

Fig. 12 Sign convention for bending structural loads.

probability density function is a truncated Gaussian function whose integral does not exceed 0.25. This portion corresponds to the probability density function whose integral is Eq. (36) of Ref. 2 repeated in the following. It may be assumed that the quasi-static structural loads will be similarly distributed.

$$P_{SJO} = \tfrac{1}{4}\left[\operatorname{erfc}\frac{h_{ci}}{\sqrt{2}\sigma_i}\right]\left[\operatorname{erfc}\frac{v_{si}}{\sqrt{2}\sigma_i'}\right] \tag{18}$$

The structural dynamic component of slamming force corresponds to the situation where the previous nonGaussian random process is the input to the structural dynamic system having small damping, which is a very narrow band oscillator and the craft acts as a "structural filter." For such a situation, according to Miles and Thomson [11] (Chap. 11, p. 11-10), the output random process will tend toward Gaussian, whatever the probability distribution of the input. This result follows directly from the "central limit theorem" of probability theory. Thus, the structural dynamic loads component of slamming is considered to be Gaussian distributed.

A similar situation to structural dynamic components of load is assumed regarding the seakeeping component of loads. This, too, will be Gaussian distributed because the shear and bending moment frequency responses are also reasonably narrow band, especially due to the narrowness of the pitch and roll frequency responses.

In view of the previous assumptions, the following rationale has been developed regarding the combination of load components: 1) Conservatively, the static and 1/50 hr quasi-static slamming components of structural loading are directly summed. 2) The structural dynamic component due to slamming and the seakeeping or terrain induced loads are each Gaussian distributed. Also, it may be shown that because of the narrow band filtering effects due to seakeeping dynamics and hull structural dynamics and because of wide separation of seakeeping and structural dynamic frequencies, these two components tend to be uncorrelated. It follows from probability theory,[12] (Chap. 6, Art. 6.4; Chap. 7, Art. 7.5) that their variances may be directly summed. Thus the total variance (or variance of their sum) is the sum of variances

$$\sigma_t^2 = \sigma_{\text{s.d.}}^2 + \sigma_{\text{sea}}^2$$

where σ_t = rms combined structural load; $\sigma_{\text{s.d.}}$ = rms structural dynamic slamming structural load; σ_{sea} = rms seakeeping or terrain induced structural load.

3) Conservatively, the results of pt. (1) are directly summed to the results of pt. (2) for a 1/50 hr frequency of occurrence.

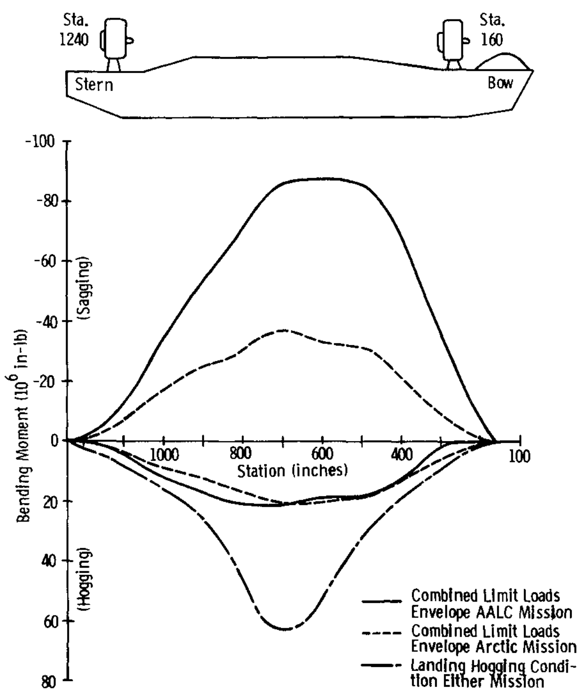

Fig. 13 Comparison of combined limit longitudinal bending moment envelopes for AALC mission and arctic mission.

This is based on the theory that the 1/50 hr occurrence probability level implies large wave conditions which in turn are likely to induce slamming, so that the simultaneous occurrence of large seakeeping loads and large slamming loads should not be discounted.

The static loads and quasi-static slamming loads are unidirectional. The results of pt. (2) are equally likely to be positive or negative. Thus, two loading conditions exist, and represent the sum and difference of the results of pts. (1) and (2). For those 2 loading conditions, one will produce tension in the upper fibers for bending and the other will produce compression. These 2 conditions are designated as "hogging" and "sagging," respectively (Fig. 12).

All of the loads were combined in the same manner for every longitudinal or transverse station of the craft. For transverse loads, no slamming components were considered. This is approximately the case according to Ref. 8.

For the purpose of analysis for seakeeping or terrain induced loads, only the longitudinal bending moment and torsion moment, together with the transverse (lateral) bending moment, all at the craft midsection, were utilized to establish long-term trends. The actual spacewise loads distribution for the 1/50 hr frequency of occurrence were then obtained by applying appropriate K_σ factors to the rms dynamic load distributions for the most frequent environmental conditions.

Application

Using the described rational dynamic loads methods and rationale for combining loads, the analysis of a nominal 150 ton ACV was conducted for the two previously described missions: 1) the Amphibious Assault Landing Craft (AALC) mission and 2) the Arctic SEV (ASEV) mission. A comparison for the two missions, of some of the resulting combined loads consisting of longitudinal bending moment and longitudinal torsional moment, are shown as envelope distributions in Figs. 13 and 14. Also shown on the bending moment distributions are distributions corresponding to the deterministic loading conditions of dynamic landing for either hogging or sagging whichever is most severe. This deterministic loading condition corresponds to the impact produced by suddenly going off-cushion in such a way as to produce a peak 2 g's of heave impact deceleration for contact

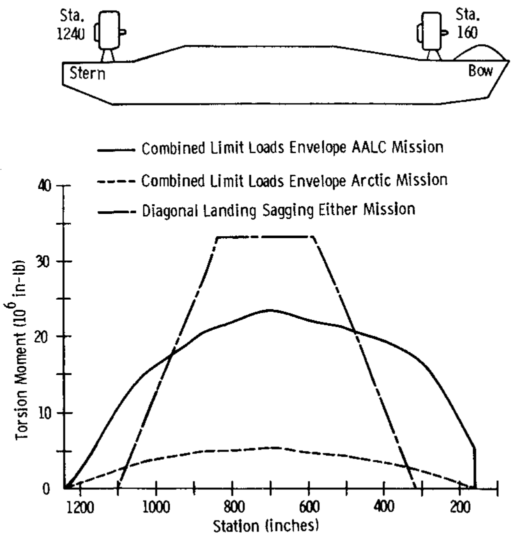

Fig. 14 Comparison of combined limit longitudinal torsion moment envelopes for AALC mission and arctic mission.

at both bow and stern (sagging) or for contact at the craft mid-section (hogging). Only the longitudinal torsion moment distribution for diagonal sagging are shown.

It is readily seen, from inspection of the curves of Figs. 13 and 14 that the AALC mission produced a significantly greater limit load envelope than did the ASEV mission. In fact the loads resulting from the AALC mission are over twice as severe as the loads resulting from the ASEV mission. This is because bow slamming was significant for the ASEV mission. The important parameter governing this effect is the much higher skirt used for the ASEV mission (9 ft as opposed to 5 ft for the AALC mission).

Conclusions

The significant conclusions that can be made regarding the rational loads analysis procedure developed in this paper are: 1) A rational probabilistic approach of air cushion vehicle static and dynamic loads analysis based on specified mission requirements utilizing an acceptable frequency of occurrence criterion is demonstrated.

2) The influence of differing mission requirements on the determination of limit loads for the same vehicle are rationally determined quantitatively. In the application of the method to the AALC and ASEV missions, it was found that significant differences in limit loads were expected between the two missions, the ASEV mission requiring less than half the load capability of the AALC mission.

3) Both quasi-static and structural dynamic components of slamming are combined with static and random dynamic components to determine limit loads for air cushion vehicles. These loads may be used in the form of loads envelopes and compared with other, deterministic loads (such as dynamic landing loads) to establish design loads.

Future Work

One important feature of the analytical procedure presented which was not mentioned, is the need for obtaining a measure of loads validation through model or full scale tests of SES/ACV craft. Although the motion (heave, pitch, roll) parameters have been validated to a certain extent by model tests,[2] it would be desirable to establish correlation between prediction and tests of, at least, mid-section bending moment. The method of combining slamming loads (both quasi-static and structural dynamic) was necessitated by the fact that they were calculated separately. A mathematical model should be developed which would account for slamming and seaway induced rigid body and structural dynamic loading considered together because they are measured together in practice. Finally, the limitations of the present procedure do not provide the detail designer with balanced external loads distributions that can be described as design loads for detail structural analysis because they are envelope conditions. Balanced load distribution cases descriptive of the limit combined loads are necessary for adequate detail stress analysis in a structural finite element mathematical model.

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